[This question paper contains 5 printed pages]

Your Roll No. ⑨ : .....2.0.1.9.......

Sl. No. of Q. Paper : **7462** J

Unique Paper Code : 32351102 - OC

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Algebra

Semester : I

Time : 3 Hours   *Maximum Marks : 75*

Instructions for Candidates :

  (i) Write your Roll No. on the top immediately on receipt of this question paper.

  (ii) Attempt any **two** parts from each questions.

  (iii) **All** questions are compulsory.

1. (a) Find the polar representation for the complex number   6

$$z = 1 - \text{Cos}\,a + i\,\text{Sin}\,a, \quad a \in [0, 2\pi)$$

  (b) Solve the equation $(2 - 3i)z^6 + 1 + 5i = 0.$ 6

  (c) Compute $z^n + \dfrac{1}{z^n}$, if $z + \dfrac{1}{z} = \sqrt{3}$.   6

P.T.O.

2. (a) Define ~ on $\mathbb{Z}$ by a~b if and only if $2a + 3b = 5n$ for some integer n. Prove that ~ defines an equivalence relation on $\mathbb{Z}$. 6

(b) Define $f: \mathbb{Z} \to \mathbb{Z}$ by $f(x) = 3x^3 - x$.

   (i) Is f one-to-one ?

   (ii) Is f onto ?

   Justify each answer. 6

(c) Show that the open intervals $(0, 1)$ and $(1, 2)$ have the same cardinality. 6

3. (a) Define relatively prime integers. Show that 17,369 and 5,472 are relatively prime. Hence, find integers x and y such that $17369x + 5472y = 1$. 6

(b) (i) Show that $3^6 = 1 \pmod 7$ and hence evaluate $3^{360} \pmod 7$.

   (ii) Find all integers $x \pmod{12}$ that satisfy $9x \equiv 3 \pmod{12}$. 6

(c) Use the Principle of Mathematical Induction to prove $2^{2n} - 1$ is divisible by 3, $\forall n \geq 1$. 6

4. (a) Write the solution set of the given system of equations in parametric vector form. 6.5

$$x_1 + 3x_2 + x_3 = 1$$

$$-4x_1 - 9x_2 + 2x_3 = -1$$

$$-3x_2 - 6x_3 = -3$$

(b) Let $A = \begin{pmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{pmatrix}$. Show that the equation $Ax = b$ may not be consistent for every $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Also describe the set of all vectors b for which $Ax = b$ is consistent.

6.5

(c) Determine h and k such that the solution set of the given system. 6.5

$x_1 + 3x_2 = k$

$4x_1 + h\, x_2 = b$

(i) is empty.

(ii) contains a unique solution.

(iii) contains infinitely many solutions.

5. (a) Boron sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation is $B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$.

Balance the chemical equation using the vector equation approach. 6.5

(b) Find the value of h for which the following vectors are linearly dependent. Also find a linear dependence relation among them. 6.5

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ h \end{bmatrix}$$

(c) A linear transformation $T : R^2 \rightarrow R^2$ first performs a vertical shear that maps e_1 into $e_2 - 2e_1$, leaves the vector e_2 unchanged and then reflects point through the line $x_2 = x_1$
   (i) Find Matrix A such that $T(x) = Ax$, $x \in R^2$.

   (ii) Find x such that $T(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. 6.5

6. (a) Given :

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$

   (i) Show that the matrix A is row equivalent to I_3.
   (ii) Find inverse of A and hence find inverse of A^T. 6.5

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(b) Find a basis for column space for the matrix A

6.5

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 & -9 \\ -2 & -2 & 2 & -8 & 2 \\ 2 & 3 & 0 & 7 & 1 \\ 3 & 4 & -1 & 11 & -8 \end{bmatrix}$$

(c) Is $\lambda = 4$ an eigen value of the matrix A ?

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$$

If so, find eigen space of A corresponding to eigen value $\lambda = 4$

6.5

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